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Quantum information through Hall qubit in frustrated QHE system

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The presence of disorder in frustrated spin system offers conflict in the spin ordering. The hopping electron develops a different kind of Berry connection in the frustrated spin system. The Quantum Hall state for the lowest Landau level at $\nu = 1$ is highly frustrated. They are the singlet states identified as the Hall qubit, the building block of other higher IQHE/FQHE states at different filling factors. In this paper we have studied the Physics behind the Hall qubit formed by entanglement of two qubits where one qubit is rotating in the field of the other with Berry phase or Aharonov-Bohm phase. With the condition of concurrence for maximum entanglement, a proper ratio between the fluxes of the entangling qubits has been evaluated. Interplay between disordered and ordered phase takes place at particular condition in QHE system.

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I. INTRODUCTION

The theory of quantized hall effect poses a problem as the impurity scattering theory and its re-normalization group extension suggest that all the state in the band are localized [1]. However, quantized Hall conductivity requires the existence of at least one extended state per Landau band of two-dimensional electron gas in a sufficiently strong magnetic field [2,3]. This peculiar situation has been analyzed by Levine,Libby and Pruisken [4] who have suggested that the extended states with localized band tails are due to topological nontrivial transformations. The ground state of elementary excitations in the FQHE is quite successful in describing the filling fractions $\nu = 1/m$, with m=odd integer by Laughlin [2]. An extension of Laughlin's theory is required to explain the experimentally observed higher filling fractions of the form p/q. Haldane [5] first gave a deeper insight into these hierarchical states by explaining that ground state of each level of hierarchy becomes a condensate of elementary excitations of the previous one. These are quasi-particles obeying fractional statistics. Wilczek [6], considered these quasi-particles as charged particles tied to flux tubes in two dimensions. Jain gave [7] an alternative approach of hierarchies by mentioning "FQHE of fermions are nothing but IQHE of composite fermions.

This characteristic feature of QHE could be realized through quantization process. Quantization plays an important role to realize the individual Physics behind IQHE and FQHE where latter is observed in extremely clean samples and impurity is found to be an essential ingredient for pronounced IQHE [8]. The strong magnetic field is responsible to quantize the Hall particle and to associate topological features acquired through Berry phase [9]. From the view point of Berry phase and aspect of angular momentum we have build a common origin between IQHE and FQHE [10] and also studied the quasi-particles in Hierarchies and the shift vectors of FQHE states [11].

The quasi-particles interwind and transport information through long-range quantum correlations by Aharanov-Bohm interactions in FQHE [12]. The transport of a charge around a flux is equivalent to the Aharanov-Bohm (AB) phase that is the very cause of quantum entanglement between particles.The quantum gates that convey topological transformation is known as topological gates. These gates are advantageous for their immunity and resistive power against local disturbances [13]. This indicates that a quantum mechanical state could carry its memory during its spatial variation and the influence of Berry phase (BP) on an entangled state could be linked up with the local observations of spins. It is natural to think that there is an intimate connection between Berry phase and entanglement when one-qubit rotates in presence of circulating magnetic field around another to form a two qubit under spin echo method [14]. The essence of quantum entanglement has been found in studying the Hall qubits specially in the lowest Landau level $\nu = 1$ [15]. This idea of entanglement has been successfully extended [16] for higher filling factors through preparation and rotation of Hall qubits having A-B interactions.

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The electrons in the Quantum Hall systems are so highly frustrated that the ground state is an extremely entangled state. Frustration is developed due to interaction between the spins that are in conflict with [17] each other due to some quenched disorder. This frustration is essential to stabilize the chiral spin states that are quantum liquid having an energy gap and characterized by the concept of topological order [18]. The local order parameter having topological stability can be realized through Berry phase in frustrated spin system [19]. In absence of disorder or in an un-frustrated system the developed Berry phase is an usual solid angle [14]. A different Berry connection [20] will be acquired by a quantized spinor in the frustrated system. In the next section-II we first reviewed the appearance of BP and AB phases in QHE. Order and disordered BPs are discussed in section III. After that the above physics of frustration will be studied respectively in the lowest Landau level of QHE in section IV and for FQHE in section V with the interplay of ordered and disordered topological phase.

II. TOPOLOGICAL ASPECT OF HALL FLUID AND COMPOSITE FERMION THEORY IN FQHE THROUGH BERRY PHASE AND AHARONOV-BOHM PHASE

In the Hall fluid the statistical interaction takes the most significant role. Being long ranged it is treated non-perturbatively. A non-dynamical gauge field A_μ is associated with the flux which in $2+1$ dimension is the very cause of the appearance of Chern-Simon term in the Lagrangian.

$$L_{CS} = \frac{\mu}{2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda \tag{1}$$

This licenses a conservation of topological current J_μ which include a topological invariant term in the (2+1) dimension [21]

$$H = \frac{\theta}{2\pi}\int d^3x A_\mu J^\mu \tag{2}$$

in the action.In fact it is the Hopf invariant describing basic maps of S^3 to S^2. If ρ denotes a four dimensional index then we find [22]

$$\partial_\rho\epsilon^{\rho\mu\nu\lambda}A_\mu F_{\mu\nu} = \frac{1}{2}\epsilon^{\rho\mu\nu\lambda}F_{\rho\mu}F_{\nu\lambda} \tag{3}$$

which connects the Hopf invariant with chiral anomaly. This Hopf term plays a role somewhat similar to the role played by the Wess-Zumino interaction in connection with $3+1$ dim. Skyrmion term.

In (3+1) dimensions, the 2D Hall surface is considered following Haldane [5] as a boundary surface of a 3D sphere, having radius R in a radial (monopole) magnetic field $B = \hbar S/eR^2 (> 0)$. This $2S = N_\phi$ is an integer which defines the total number of magnetic flux through the surface. For the parent state $\nu = 1/m$ the total flux is $S = \frac{1}{2}m(N-1)$. The field strength S in the first level hierarchy is

$$S(N, m \pm p) = \frac{1}{2}m(N-1) \pm \frac{1}{2}(\frac{N}{p}+1)$$

which is formed when p (p= even integer) excitations are added in the parent state $\nu = \frac{1}{m}$. These show that the filling factors for hierarchical state satisfy a slight complicated relation.

$$2S = N\phi = \nu^{-1}N - \mathcal{S}$$

For a $\nu = \frac{1}{m}$ parent state this *shift* is simply $\mathcal{S} = 2(n-1)+m$ having orbital spin $s = n-1+\frac{m}{2}$ that is associated with the orbital angular momentum in cyclotron motion. In the effective theory, this introduction of *shift* leads to a modification of the Lagrangian in equation (1). The appearance of *shift* in the hierarchies of FQHE is nontrivial [23].

$$\mathcal{L} = 1/4\pi(KB\epsilon\partial B + 2Ae\epsilon\partial B + 2Cs\epsilon\partial B) \tag{4}$$

where the second term is the electromagnetic coupling and the third one is the coupling to the curvature of space.

The quantization of Fermi field can be achieved assuming anisotropy in the internal space through the introduction of direction vector as an internal variable at each space-time point [11]. The opposite orientations of the direction vector correspond to particle and antiparticle. Incorporation of spinorial variables $\theta(\bar{\theta})$ in the coordinate result the enlargement of manifold from S^2 to S^3.This helps us to consider a relativistic quantum particle as an extended one, where the extension involves gauge degrees of freedom. As a result the position and momentum variables of a quantized particle becomes

$$Q_\mu = i\left(\frac{\partial}{\partial p_\mu} + A_\mu\right), \qquad P_\mu = i\left(\frac{\partial}{\partial q_\mu} + \tilde{A}_\mu\right) \tag{5}$$

where q_μ and p_μ are related to the position and momentum coordinates in the sharp point limit and $A_\mu(\tilde{A_\mu})$ are non-Abelian matrix valued gauge fields belonging to the group $SL(2C)$. In fact the quantization of Hall particles is the indication of Quantum Hall effect involving gauge theoretic extension of coordinate by $A_\mu \in SL(2C)$ which is visualized through the field strength $\tilde{F_{\mu\nu}}$.

The quasiparticles in hierarchy levels are formed when additional fluxes are attached with quantized particles.Apart from the internal extension, the external strong magnetic field induces gauge extensions $B_\mu \in SL(2C)$ through the gauge field $F_{\mu\nu}$.In the language of differential geometry these two gauges act as two fibres at each particle points of the base space S^2. The effective theory of the Hall fluid (Abelian) can be accurately presented if not only the two vortices but also their interactions are taken into account. In presence of strong external magnetic field, the chiral symmetry breaking of composite fermion associated with internal and external gauge fields $F_{\mu\nu}$ and $\tilde{F_{\mu\nu}}$ are represented by

$$F_{\mu\nu} = \partial_\mu B_\mu - \partial_\nu B_\nu + [B_\mu, B_\nu]$$

$$\tilde{F_{\mu\nu}} = \partial_\mu A_\mu - \partial_\nu A_\nu + [A_\mu, A_\nu] \tag{6}$$

The topological Lagrangian of Hall fluid in (3+1) dimension can be described by the added Chern-Simon terms in the Lagrangian through the anomaly [11]

$$\mathcal{L} = -\frac{\theta}{16\pi^2} Tr^* F_{\mu\nu} F_{\mu\nu} - \frac{\theta'}{16\pi^2} Tr^* F_{\mu\nu} \tilde{F}_{\mu\nu} - \frac{\theta''}{16\pi^2} Tr^* \tilde{F}_{\mu\nu} \tilde{F}_{\mu\nu} \tag{7}$$

Here every term corresponds to a total divergence of a topological quantity, known as Chern-Simons secondary characteristics class defined by

$$\Omega^\mu{}_e = -\frac{1}{16\pi^2 \epsilon_{\mu\nu\alpha\beta}} Tr[B_\nu F_{\alpha\beta} - 2/3(B_\nu B_\alpha B_\beta)] \tag{8}$$

$$\tilde{\Omega}^\mu = -\frac{1}{16\pi^2 \epsilon_{\mu\nu\alpha\beta}} Tr[C_\nu F_{\alpha\beta} - 2/3(C_\nu B_\alpha B_\beta)] \tag{9}$$

$$\Omega^\mu{}_i = -\frac{1}{16\pi^2 \epsilon_{\mu\nu\alpha\beta}} Tr[C_\nu \tilde{F}_{\alpha\beta} - 2/3(C_\nu C_\alpha C_\beta)] \tag{10}$$

In particular the θ term in the Lagrangian leads to vortex line and the corresponding gauge field acts like a magnetic field. Assuming a particular choice of coupling $\theta = \theta' = \theta$" in the Lagrangian the topological part of the action in (3+1) dimension become

$$W_\theta = 2(\mu_e + \mu_i + \tilde{\mu})\theta \tag{11}$$

where μ_e, μ_i and $\tilde{\mu}$ are the corresponding magnetic charges which are connected with the respective charges through the Dirac quantization condition and Pontryagin density.

$$2\mu = q = \int \partial_\mu \Omega_\mu d^4x \tag{12}$$

It gives rise the topological phase of Berry on the parallel transport over a closed path of a Hall hierarchy state.

$$\phi_B = \pi W_\theta = 2\pi\tilde{\mu}_{eff}\theta = 2\pi(\mu_{eff} + \tilde{\mu})\theta = 2\pi(\mu_e + \mu_i + \tilde{\mu})\theta \tag{13}$$

Here the first term μ_e is associated with Berry phase factor of Hall particle due to external magnetic field μ_e, the second term μ_i gives rise to the inherent Berry phase factor associated with the chiral anomaly of a free electron (in absence of an external magnetic field) and the third one effectively relates the coupling of the external field with the internal one which give rise the phase factor $\tilde{\mu}$. This μ_{eff} actually visualizes the filling factor of the composite fermion through the relation $\nu = \frac{n}{2\mu_{eff}}$ where n denotes the nth Landau level in which the particle resides. In fact this μ_{eff} satisfies the Dirac quantization condition

$$e'\mu_{eff} = \frac{n}{2} \tag{14}$$

showing that each quasi particle in the n^{th} Landau level having charge e' behaves as a composite fermion. It will behave as fermion in the ground state following the Dirac condition

$$\tilde{e\mu} = \pm 1/2 \tag{15}$$

that arises when

$$\tilde{e}(\mu_{eff} - \frac{n \pm 1}{2}) = \pm \frac{1}{2} \tag{16}$$

This implies that $(n \pm 1)/2$ is the magnetic strength μ of the added quanta whose removal makes the composite fermion in the higher Landau levels to behave as fermion in the ground state. Here for $\mu = \pm 1/2, \pm 3/2, ...$ the quanta behave like fermion and $\mu = \pm 1, \pm 2, ...$ it shows bosonic behavior. It has realized that this change of magnetic charge as $\tilde{\mu}$ can be visualized [11] through *shift* $\mathcal{S}$ by the relation

$$2\tilde{\mu} = \mathcal{S} = 2\mu_{eff} - (n \pm 1) = \frac{n}{\nu} - (n \pm 1) \tag{17}$$

where $n = 1, 2, 3..$ denotes the hierarchy levels. Our picture shows that a transport of composite particle in the Hall fluid moving in a circular orbit will be quantized through its acquirance of the Berry's topological phase.

$$\phi_B = \pi\theta(2\mu_{eff} + \mathcal{S}) \tag{18}$$

Conceptually the appearance of this *shift* quantum number $\mathcal{S}$, in the topological phase of quasi-particle is obvious, since the coupling between the two gauges(that act as fibres) with the curvature is prominent during parallel transport over a closed path. With this view we have studied [24] the role of *shift* in the relative AB type phase as the composite fermion and the additional quanta encircles each other.

Composite fermion theory in FQHE and A-B type phase :

In the composite fermion theory of Quantum Hall effect the qubits are equivalent to the fluxes attached with the charged particles. When an electron is attached with a magnetic flux, its statistics changes and it is transformed into a boson.These bosons condense to form cluster which is coupled with the residual fermion or boson (composed of two fermions). Indeed the residual boson or fermion will undergo a statistical interaction tied to a geometric Berry phase effect that winds the phase of the particles as it encircles the vortices.Indeed as two vortices cannot be brought very close to each other, there will be a hard core repulsion in the system which accounts for the incompressibility of the Quantum Hall fluid.

These non-commuting fluxes have their own interesting Aharonov-Bohm interactions. As the quasi-particles encircles another in their way of topological transport, the Aharonov-Bhom type phase and statistical phase are developed. Following a generalization of Pauli exclusion principle, Haldane [25] pointed out that the quasi-particles carrying flux ϕ_α and charge q_β orbiting around another object carrying flux ϕ_β and charge q_β has the relative statistical phase $\theta_{\alpha\beta}$

$$exp(i\theta_{\alpha\beta}) = exp \pm i\pi(g_{\alpha\beta} + g_{\beta\alpha}) \tag{19}$$

where $g_{\alpha\beta} = -q_\alpha\phi_\beta$.

With this view we have recently shown [26] that when two non-identical composite fermions residing in two consecutive Landau levels in FQHE encircle each other, the relative Aharonov-Bhom (AB) type phase is developed. As the quasi-particles advance towards the edge of FQHE in a similar circular way,the developed current [24] should have a connection with this AB type phase through Berry's topological phase.

We assume the transfer of the composite particle from the inner edge in the n^th Landau level having filling factor ν_n picking up even integral $(2m)$ of flux ν_1 through the bulk of QH system and forming a new composite particle in the $(n+1)^th$ Landau level of the outer edge. The filling factor and the associated flux of the effective particle becomes

$$\nu_{eff} = \frac{n+1}{\mu_{eff}} \tag{20}$$

which can be considered as

$$\mu_{eff} = 2m\mu_1 + \mu_n = 2m(\frac{N-1}{2}) + \frac{N-n^2}{2n} \tag{21}$$

Here statistical interaction takes place between the composite particle of the inner edge and outer edge which is responsible for propagation of current [24]. We further assume that path of the particles do not intersect each other. Encircling one type of fluxes around another in the consecutive Landau level generate the relative AB type phase developed by their fluxes and charges as

$$\phi_s = \exp \pm \frac{i\pi}{2}(q_n\mu_{eff} + q_{eff}\mu_n) \tag{22}$$

In use of the Dirac quantization relation the charges of the quasiparticles are replaced by the Landau filling factor in eq.(22)

$$\phi_s = \exp \pm \frac{i\pi}{2}(\nu_n\mu_{eff} + \nu_{eff}\mu_n) \tag{23}$$

In hierarchies of FQHE the effective flux ν_{eff} in outer edge is equivalent to combined effect of even integral(2m) of flux μ_1 in bulk and transporting flux μ_n of inner edge

$$\phi_s = \exp \pm \frac{i\pi}{2}(\frac{n}{\mu_n}(2m\mu_1 + \mu_n) + \frac{(n+1)\mu_n}{(2m\mu_1 + \mu_n)} \tag{24}$$

$$\phi_s \cong \exp \pm \frac{i\pi}{2}[\frac{n}{2}(2m\frac{\mu_1}{\mu_n} + 1) + \frac{n+1}{2}(1 - 2m\frac{\mu_1}{\mu_n})] \tag{25}$$

$$\phi_s \cong \exp \pm \frac{i\pi}{2}[(n + \frac{1}{2}) - m\frac{\mu_1}{\mu_n}] \tag{26}$$

This relative AB type phase factor has relationship with *shift* quantum number. It can be be commented from the above equation that irrespective of μ_1 and μ_n being fermionic or bosonic flux, the phase factor depends upon the number of particles-N,the Landau level-n, and the odd integer-m that is the inverse of parent filling factor $\nu = 1/m$ by the following expression.

$$\phi_s \cong \exp \pm \frac{i\pi}{2}[(n + \frac{1}{2}) - \frac{mn(N-1)}{(N-n^2)}] \tag{27}$$

Recently it has been found that fractional statistics play an important role in topological transformation in connection with Quantum computation [27]. Also Kane [28] showed that statistical phase by combining AB effect can be used in noise measurement. The result we find fully support these views. There is an intimate connection between AB phase with Berry phase (BP). The BP for non-degenerate state is trivial. For degenerate state this BP becomes of matrix valued. In the next section, we will discuss about the non-trivial BP in presence of disorder.

III. ORDERED AND DISORDERED BERRY PHASES

The relationship of Berry Phase (BP) with topological quantum number is independent of any approximation scheme. In general BP is the solid angle subtended by a quantum particle driven by parameter that can be seen by considering the two component nature of fermion represented by spherical harmonics ${Y_l}^{m,\mu}$. For half orbital and spin angular momentum, i.e. for $l = 1/2, |m| = |\mu| = 1/2$,the two component up-spinor structure $|\uparrow\rangle$ of a quantized spin-1/2 particle becomes [14]

$$\begin{aligned} |\uparrow\rangle &= \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} {Y_{1/2}}^{1/2,1/2} \\ {Y_{1/2}}^{-1/2,1/2} \end{pmatrix} \\ &= \begin{pmatrix} \sin\frac{\theta}{2}\exp i(\phi-\chi)/2 \\ \cos\frac{\theta}{2}\exp -i(\phi+\chi)/2 \end{pmatrix} \end{aligned} \tag{28}$$

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In the language of quantum computation,a spinor can be written by use of quantum gates and an arbitrary superposition of elementary qubits $|0\rangle$ and $|1\rangle$

$$|\uparrow\rangle = \left(\sin\frac{\theta}{2}e^{i\phi}|0\rangle + \cos\frac{\theta}{2}|1\rangle\right)e^{-i/2(\phi+\chi)} \tag{29}$$

Similar to the coherent state approach [28], the effective lagrangian $L^{\uparrow}_{eff}$ of these states thus becomes

$$L^{\uparrow}_{eff} = <\uparrow|\nabla_t|\uparrow> = -\frac{i}{2}(\dot{\chi} + \dot{\phi}\cos\theta) \tag{30}$$

All the angular parameters being time dependent, ∇_t is the derivative with respect to time.

The related geometrical phase of a quantized spinor could be obtained from the action integral over a closed path [28].

$$\begin{aligned} \gamma_{\uparrow} &= i\int L^{\uparrow}_{eff}dt && (31)\\ &= i\oint A_{\uparrow}(\lambda)d\lambda && (32)\\ &= \frac{1}{2}(\oint d\chi + \cos\theta\oint d\phi) && (33)\\ &= \pi(1+\cos\theta) && (34) \end{aligned}$$

This shows that for quantized spinor the Berry Phase is a solid angle subtended about the quantization axis. In a similar manner the parallel transport of this down-spinor over the closed path will generate the Berry phase by the solid angle of opposite chirality.

$$\gamma_{\downarrow} = -\pi(1+\cos\theta) \tag{35}$$

The fermionic or the antifermionic nature of the two spinors (up/down) can be identified by the maximum and minimum values of topological phases $\gamma_{\uparrow/\downarrow} = \pm\pi$, 0 for the respective θ values $\pi/2$ and π. These findings is similar with that of Hatsugai [29] who pointed out that the frustrated spin systems can be well classified by the respective quantized Berry phase through 0 and π as the local topological order.

In absence of local frustration the parallel transport of the quantized spinors develops the Berry phase visualized by the solid angles as in eq.(34) or (35) where the final direction of spinor merges with the initial completing a closed path. But in frustrated spin system,the situation is different where the spinor does not trace a closed path. Thus the different kind of Berry connection developed in frustrated spin system will be investigated further based on the following situation.

If an electron coupled to a spin hops from site i to site j then Ohgushi,Murakami and Nagaosa [30] pointed out that the spin wave function is effectively

$$|C_i\rangle = t\left(e^{ib_i}\cos\frac{\theta_i}{2}, e^{i(b_i+\phi_i)}\sin\frac{\theta_i}{2}\right) \tag{36}$$

Here t denotes the hopping integral that acts as the coupling between the neighboring spins. The overall phase b_i visualizing the gauge degree of freedom does not appear as physical quantity. Moreover, following Anderson's analysis [31] the effective transfer integral t_{ij} is given by

$$\begin{aligned} t_{ij} &= t\langle C_i|C_j\rangle \\ &= te^{i(b_j-b_i)}\left(\cos\frac{\theta_i}{2}\cos\frac{\theta_j}{2} + e^{i(\phi_j-\phi_i)}\sin\frac{\theta_i}{2}\sin\frac{\theta_j}{2}\right) \\ &= te^{ia_{ij}}\cos\frac{\theta_{ij}}{2} && (37) \end{aligned}$$

where θ_{ij} is the intermediate angle between the two spin vectors $\vec{S_i}$ and $\vec{S_j}$. The phase a_{ij} is the vector potential, generated by the spins, measuring the spin chirality in the context of quantum spin liquid where the spins fluctuate quantum mechanically.

Motivated by the above ideas, we will now proceed to find out the other kind of Berry connection developed by a quantized spinor in eq.(28) on a frustrated sphere. A frustrated sphere can be realized as an extended sphere with three parameters θ,ϕ and χ where each point on the surface is occupied by quantized spinor or local disorder. These disorders are responsible for producing randomness in the spin degrees of freedom. As the quantized up-spinor hops from the site i to the site j, the transfer integral t_{ij} becomes

$$\langle\uparrow_i|\uparrow_j\rangle = e^{i/2(\phi_i-\phi_j)}.e^{i/2(\chi_i-\chi_j)}$$
$$\left(\cos\frac{\theta_i}{2}\cos\frac{\theta_j}{2} + e^{i(\phi_j-\phi_i)}\sin\frac{\theta_i}{2}\sin\frac{\theta_j}{2}\right) \tag{38}$$

which is product of the intermediate angle θ_{ij} and difference of inclination of helicity of the quantized spinor as it moves from site i to j in frustrated spin glass.

$$\langle\uparrow_i|\uparrow_j\rangle = e^{i\frac{(\phi_i-\phi_j)}{2}} e^{i\frac{(\chi_i-\chi_j)}{2}}\cos\frac{\theta_{ij}}{2} \tag{39}$$

In 3D anisotropic space, μ, the measure of chiral anomaly [32] (non-conservation of chiral current) can take the values $\pm 1/2$ for which the helicity dependent Berry phase is $\Phi_B = e^{2i\pi\mu} = e^{\pm i\pi}$. As a scalar particle traverses a closed path with one flux quantum enclosed, the acquired phase $e^{i\pi}$ suggests the system as a fermion. In a frustrated space we consider the helicity dependent Berry phase of a two-component spinor ($\mu = 1/2$) hopping from site i to site j ($i \neq j$) as.

$$e^{i\frac{(\chi_i-\chi_j)}{2}} = e^{i\frac{\delta\chi}{2}} = e^{i\frac{a_{ij}}{2}}$$

As a result, the equation (20) becomes

$$\langle\uparrow_i|\uparrow_j\rangle = e^{i/2(\phi_i-\phi_j+a_{ij})}\cos\frac{\theta_{ij}}{2} \tag{40}$$

The uses of local gauge transformation

$$a_{ij} \longrightarrow a_{ij} + \phi_i - \phi_j \tag{41}$$

enable us to modify the transfer integral for the quantized up-spinor by

$$\langle\uparrow_i|\uparrow_j\rangle = e^{ia_{ij}/2}\cos\frac{\theta_{ij}}{2} \tag{42}$$

It has found [30] that for nearest-neighbor spin coupling $t = 1$, the total energy when optimized the above transfer integral eq.(18) for quantized up-spinor becomes similar with that of Anderson [13] and Ohgushi,Murakami and Nagaosa[31] as seen in equation (37). This comparison is justified if we identify the gauge degrees of freedom of the spinor at ith site by $b_i = -(\phi_i + \chi_i)/2$, and then the vector potential is developed from spin chirality $a_{ij} = (b_i - b_j)$ to measure the difference of inclination of the transported spinor at the two different sites.

In presence of local frustration,we realize that the quantized spinor could not trace a closed curve due to randomness of spin degree of freedom. This results the shift of the final point from the initial developing a different kind of Berry connection. The required Berry connection on the frustrated system has now been calculated considering the constancy of parameters θ and χ (having negligible variations) during cyclic motion of quantized spinor. In fact, the quantization fixes the helicity of spinor by χ and the respective initial and final site i and j can be considered on the circular arc in any hemi-sphere of constant θ but of different ϕ values. Hence,during in our model only the derivative of ϕ has been considered to evaluate the frustrated Berry connection. Thus

$$\langle\uparrow_i|\nabla_\phi|\uparrow_j\rangle = \frac{i}{2}e^{i/2(\phi_i-\phi_j)}.e^{i/2(\chi_i-\chi_j)}.$$
$$\left(\sin\frac{\theta_i}{2}\sin\frac{\theta_j}{2}e^{i(\phi_j-\phi_i)} - \cos\frac{\theta_i}{2}\cos\frac{\theta_j}{2}\right) \tag{43}$$

using eq.(38)

$$\langle\uparrow_i|\nabla_\phi|\uparrow_j\rangle = \frac{i}{2}e^{ia_{ij}/2}.$$
$$\left(\cos\frac{\theta_{ij}}{2} - 2\cos\frac{\theta_i}{2}\cos\frac{\theta_j}{2}\right) \tag{44}$$

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It is seen that the above frustrated connections measure the vector potential a_{ij}, along with the intermediate angle θ_{ij} where the randomness of spin orientations plays the crucial role without changing the polarity of spinor as it hops from the site i to site j. The intermediate angle θ_{ij} plays the key role to visualize the frustration in Berry connection. The topological phase for frustrated spinor can be written using Berry connection integrated over $0 \le \phi \le 2\pi$ only as follows

$$\Gamma^{\uparrow}{}_{F} = i \oint \langle \uparrow_i |\nabla_\phi| \uparrow_j \rangle . d\phi \tag{45}$$

The respective Berry phases [20] for both up and down spinors in frustrated spin glass system becomes

$$\Gamma^{\uparrow}{}_{F} = \pi e^{ia_{ij}/2} \left(2\cos\frac{\theta_i}{2}\cos\frac{\theta_j}{2} - \cos\frac{\theta_{ij}}{2} \right) \tag{46}$$

and

$$\Gamma^{\downarrow}{}_{F} = -\pi e^{-ia_{ij}/2} \left(2\cos\frac{\theta_i}{2}\cos\frac{\theta_j}{2} - \cos\frac{\theta_{ij}}{2} \right) \tag{47}$$

These phases depend not only on the individual angles θ_i and θ_j of the spinors but also mainly on the intermediate angle θ_{ij} and the gauge a_{ij} due to difference of spin directions.

In absence of frustration, no randomness of spin degree of freedom is seen indicating $a_{ij} = 0$. Moreover if the intermediate angle θ_{ij} is π, no spin conflict persist resulting $\cos\frac{\theta_{ij}}{2} = 0$. In this situation ideally the parallel transport of spin vectors takes place. As a result the final site of the spinor coincide with the initial leading to choose $\chi_i = \chi_j$, $\phi_i = \phi_j$ and $\theta_i = \theta_j$. The well known Berry phases

$$\gamma^{\uparrow} = \pi(1 + \cos\theta_i) \tag{48}$$

$$\gamma^{\downarrow} = -\pi(1 + \cos\theta_i)$$

are recovered in the form of usual solid angles for the respective up and down spinors. This shows that the physics of frustrated spin system becomes more transparent through Berry phase. In a frustrated system, $\cos\frac{\theta_{ij}}{2} = \pm 1$, act as a signature of two chirality that may behave also as an order parameter in the system. It is important to highlight that even in disordered quantum system the new kind of Berry phase could be transformed to the conventional ordered phase(solid angle) at a special condition [33]. This above physics of frustration is very nicely applied in Quantum Hall system where disorder plays an important role for localization. In the next section it could be seen how this frustrated Berry phase is responsible in appearance of matrix form of non-abelian BP in Quantum Hall effect.

IV. HALL QUBIT FORMATION THROUGH BERRY PHASE IN FRUSTRATED INTEGER QUANTUM HALL STATE

In the Quantum Hall system the external magnetic field in presence of disorder introduces a frustration. We have considered a two-dimensional frustrated electron gas of N particles on the surface of a three dimensional sphere of large radius R in a strong radial (monopole) magnetic field. In such a 3D anisotropic space with the spirit of Haldane [5] we can construct the N-particle wave-function of Hall excitations at the occupations $\nu = 1/m$, m being an odd integer. Evidently for odd(even) m we will have respective fermionic (bosonic) state. Following Haldane this $m = J_{ij} = J_i + J_j$ is the two particle angular momentum equivalent to $m = \mu_i + \mu_j = 2\mu$ (when $i = j$) when the angular momentum in the anisotropic space is given by $\mathbf{J} = \mathbf{rxp} - \mu\mathbf{r}$. From the description of a qubit as two component up spinor [11] $|\uparrow\rangle = \binom{u}{v}$, where u ,v are spherical harmonics ${Y_l}^{m,\mu}$ we can construct the N particles wave function of Hall states for half orbital angular momentum, $l = 1/2$, $|m| = |\mu| = 1/2$.

$$\Psi_{N_\uparrow}{}^{(m)} = \prod (u_i v_j - u_j v_i)^m \tag{49}$$

The Hall state with opposite polarization can be constructed similarly by using the down spinor $|\downarrow\rangle = \binom{\tilde{v}}{\tilde{u}}$

$$\Psi_{N_\downarrow}{}^{(m)} = \prod (\tilde{u}_i \tilde{v}_j - \tilde{u}_j \tilde{v}_i)^m \tag{50}$$

Here the two states $\Psi_{N_\uparrow}{}^{(m)}$ and $\Psi_{N_\downarrow}{}^{(m)}$ belong to the same parent filling factor but with opposite polarization of the spinors.

It could be pointed out that as Hall effect is a topological transportation,the states could be realized to form through Berry phase by spin echo method. In the construction of two qubit through rotation of one qubit (spin 1/2) in the vicinity of another Berry phase plays the key role where dynamical phase eliminate. Incorporating the spin-echo [33] for half period we find the antisymmetric Bell's state after one cycle ($t = \tau$),

$$|\Psi_-(t=\tau)\rangle = \frac{1}{\sqrt{2}}(e^{i\gamma_\uparrow}|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - e^{-i\gamma_\uparrow}|\downarrow\rangle_1 \otimes |\uparrow\rangle_2) \tag{51}$$

and symmetric state becomes

$$|\Psi_+(t=\tau)\rangle = \frac{1}{\sqrt{2}}(e^{-i\gamma_\uparrow}|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 + e^{i\gamma_\uparrow}|\downarrow\rangle_1 \otimes |\uparrow\rangle_2) \tag{52}$$

where $\gamma_\downarrow = -\gamma_\uparrow = -\gamma$. It may be noted that for $|\mu_\uparrow| = \frac{1}{2}(1 - cos\theta)$.

Splitting up these above two eqs.(51) and (52) into the combination of initial symmetric and antisymmetric states we have after rearranging

$$|\Psi_+\rangle_\tau = \cos\gamma\,|\Psi_+\rangle_0 - i\sin\gamma\,|\Psi_-\rangle_0 \tag{53}$$
$$|\Psi_-\rangle_\tau = i\sin\gamma\,|\Psi_+\rangle_0 + \cos\gamma\,|\Psi_-\rangle_0 \tag{54}$$

the matrix Berry phase-Σ as the doublet rotated from $t = 0$ to $t = \tau$.

$$\begin{pmatrix} |\Psi_+\rangle \\ |\Psi_-\rangle \end{pmatrix}_\tau = \Sigma \begin{pmatrix} |\Psi_+\rangle \\ |\Psi_-\rangle \end{pmatrix}_0 \tag{55}$$

$$\Sigma = \begin{pmatrix} \cos\gamma & -i\sin\gamma \\ i\sin\gamma & \cos\gamma \end{pmatrix} = \cos 2\gamma \tag{56}$$

This non-abelian matrix Berry phase Σ is developed [15] from the abelian Berry phase γ. For $\gamma = 0$ there is symmetric rotation of states, but for $\gamma = \pi$ the return is antisymmetric as the values of Σ=I and -I (where I=identity matrix) respectively.

The above states are grouped into a family depending on the value of m. With $m = 3$ the states are the same family of the Laughlin $\nu = 1/3$ state [2]. It seems that for LLL $\nu = 1$, IQHE state $\Phi_1(z)$

$$\Phi_1(z) = (u_i v_j - u_j v_i) \tag{57}$$

is the basic building block for constructing any other IQHE/FQHE state. The lowest level Hall state $\Phi_1(z)$ is identified as Hall qubit which is two-qubit singlet state formed by a pair of one qubit states where one qubit is rotating in vicinity of other with Berry phase. In the light of Jain [7] "FQHE of fermions is the IQHE of composite fermions" we may point out that any FQHE state can be expressed in terms of Hall qubit, the IQHE state of LLL.

There is a deep analogy between FQHE and superfluidity [7]. The ground state of anti-ferromagnetic Heisenberg model on a lattice introduce frustration giving rise to the resonating valence bond(RVB) states corresponding spin singlets where two nearest-neighbor bonds are allowed to resonate among themselves. It is suggested that RVB states [13], [34] is a basis of fault tolerant topological quantum computation. Since these spin singlet states forming a RVB gas is equivalent to fractional quantum Hall fluid, its description through quantum computation will be of ample interest.

This resonating valence bond(RVB) where two nearest-neighbour bonds are allowed to resonate among themselves has equivalence with entangled state of two one-qubit. The antisymmetric Hall state $\Phi_1(z)$ for $\nu = 1$ is formed [15] as one spinor at ith site rotating with Berry phase $\gamma = \pm i\pi$ in the vicinity of another at jth site captures the image of spin echo

$$|\Phi_1(z)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \quad |\downarrow\rangle_1)\begin{pmatrix} 0 & -e^{-i\pi} \\ e^{i\pi} & 0 \end{pmatrix}\begin{pmatrix} |\uparrow\rangle_2 \\ |\downarrow\rangle_2 \end{pmatrix} \tag{58}$$
$$= (|\uparrow\rangle_1 \quad |\downarrow\rangle_1)\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} |\uparrow\rangle_2 \\ |\downarrow\rangle_2 \end{pmatrix} \tag{59}$$
$$= \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \quad |\downarrow\rangle_2 - |\downarrow\rangle_1 \quad |\uparrow\rangle_2) \tag{60}$$
$$= |\Psi_-\rangle \tag{61}$$

Due to symmetry, the singlet state can be written on any basis with the same form. We can rotate the spin vector by an arbitrary angle θ with the following transformation.

$$\begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix} = \begin{pmatrix} \sin\theta e^{i\phi} & \cos\theta \\ -\cos\theta & \sin\theta e^{-i\phi} \end{pmatrix} \begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix} \tag{62}$$

The Quantum Hall systems are so highly frustrated that the ground state $\Phi_1(z)$ is an extremely entangled state visualized by the formation of antisymmetric singlet state between a pair of i, jth spinors in the Landau filling factor ($\nu = 1$).

$$\begin{aligned} \Phi_1(z) &= \begin{pmatrix} u_i & u_j \\ v_i & v_j \end{pmatrix} = (u_i v_j - u_j v_i) \\ &= (u_i \quad v_i) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} u_j \\ v_j \end{pmatrix} \end{aligned} \tag{63}$$

We identify this two qubit singlet state as Hall qubit constructed from the up-spinor shown in the previous section

$$\Phi_1(z) = \langle\uparrow_i| \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} |\uparrow_j\rangle = \langle 0| U_i\dagger \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} U_j |0\rangle \tag{64}$$

The down spinor can construct the opposite polarization of Hall qubit

$$\Phi_1(\tilde{z}) = (\tilde{u}_i \tilde{v}_j - \tilde{u}_j \tilde{v}_i) \tag{65}$$

Now these two Hall qubits eq.(63) and (65) of two opposite polarizations representing the state of same lowest Landau level $\nu = m = 1$ will automatically generate two respective non-abelian Berry connections. The Hall connection for up spinor becomes

$$B_\uparrow = \Phi_1(z)^* d\Phi_1(z) = \begin{pmatrix} {u_i}^* & {v_i}^* \\ {u_j}^* & {v_j}^* \end{pmatrix} \begin{pmatrix} du_i & du_j \\ dv_i & dv_j \end{pmatrix} \tag{66}$$

and similarly for down-spinor

$$\tilde{B_\downarrow} = \Phi_1(\tilde{z})^* d\Phi_1(\tilde{z}) = \begin{pmatrix} \tilde{u_i}^* & \tilde{v_i}^* \\ \tilde{u_j}^* & \tilde{v_j}^* \end{pmatrix} \begin{pmatrix} d\tilde{u_i} & d\tilde{u_j} \\ d\tilde{v_i} & d\tilde{v_j} \end{pmatrix} \tag{67}$$

The non-abelian nature of the connection or Berry phase on the Hall surface for the lowest Landau level LLL ($\nu = 1$) will remain if $i \neq j$.

$$\begin{aligned} B_\uparrow &= \begin{pmatrix} (u_i^* du_i + v_i^* dv_i) & (u_i^* du_j + v_i^* dv_j) \\ (u_j^* du_i + v_j^* dv_i) & (u_i^* du_i + v_i^* dv_i) \end{pmatrix} \\ &= \begin{pmatrix} \mu_i & \mu_{ij} \\ \mu_{ji} & \mu_j \end{pmatrix} \end{aligned} \tag{68}$$

This is visualizing the spin conflict during parallel transport leading to matrix Berry phase. In the light of Hwang et.al [35] our realization includes that in Quantum Hall effect this non-abelian matrix Berry phase is responsible for the charge flow by pumping. In frustrated QHE system the matrix Berry phase becomes

$$\gamma^H{}_\uparrow = \begin{pmatrix} \gamma_i & \gamma_{ij} \\ \gamma_{ji} & \gamma_j \end{pmatrix} \tag{69}$$

γ_i and γ_j are the BPs for the ith and jth spinor as seen in eq.(34) and the off-diagonal BP γ_{ij} arises due to local frustration in the spin system that is equivalent to ${\Gamma_\uparrow}^F$ of eqn.(46). Over a closed period $t = \tau$ the QHE state $\Phi_1(z)$ at $\nu = 1$ filling factor will acquire the matrix Berry phase.

$$\langle\Phi_1(z)|_\tau = e^{i\gamma^H{}_\uparrow} \langle\Phi_1(z)|_0 \tag{70}$$

Berry connection gets modified as the quantum state differ after one rotation. Usually when any state changes by

$$|\psi'\rangle = |\psi\rangle\, e^{i\Omega(c)}$$

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the corresponding changed gauge becomes

$$A_{\psi}' = A_{\psi} + id\Omega(c)$$

provided $\langle\psi|\psi\rangle = 1$. We have pointed out earlier [15] that each Quantum Hall state for a particular filling factor has its distinct Berry phase. Hence BP is constant for a filling factor. The rotation could shift the BP from ground to excited level.It could be noted here that the antisymmetric nature of FQHE states would be visualized through the rotation of singlet states. The rotation of Hall qubit by 'n' number of turns will be

$$\langle\Phi_1(z)|^{n}{}_{\tau} = e^{in\gamma^{H}{}_{\uparrow}}\langle\Phi_1(z)|^{n}{}_{0} \tag{71}$$

where $n = 1, 2, 3..$ are the natural numbers associated with the number of rotations of the singlet states. This automatically imposes the following constraint in the topological phase

$$e^{in\gamma^{H}} = e^{im\pi} = -1, \tag{72}$$

for which the Hall qubit follows

$$for \quad \langle\Phi_1(z)|_{n\tau} = -\langle\Phi_1(z)|_{0}$$

To maintain the antisymmetric nature of wave function $m = 1, 3, 5..$ need to be odd numbers. Any number of rotations of Hall qubit will lead to the matrix Berry phase as odd multiple of π so that every state remains antisymmetric. It seems that BP act as a local order parameter of states in QHE system.

$$\langle\Phi_1(z)|_{\tau}^{m\pi/\gamma} = e^{im\pi}\langle\Phi_1(z)|_{0}^{m\pi/\gamma} \tag{73}$$

In extension of our previous work [10] that the Berry phase for $\nu = 1/m$ state is $\gamma = m\pi\theta = 2\pi\mu\theta$ where θ is a coupling constant. Each rotation of Hall qubit results

$$\langle\Phi_1(z)|_{\tau} = e^{im\pi\theta}\langle\Phi_1(z)|_{0} \tag{74}$$

The above Physics makes the experimental observation of parent state in FQHE at $m = odd(3, 5, 7)$ filling factors more transparent. It also shows that the topological phase is responsible for controlling the statistics of the Hall state. In absence of frustration (disorder), the Berry phase is a diagonal matrix for γ_{ij} being zero.

$$\gamma^{H}{}_{\uparrow} = \pi(1 - \cos\theta_i)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \tag{75}$$

The non-abelian matrix Berry phase in Quantum Hall effect is originated due to the frustration offered by the magnetic field and the disorder of spins. In the absence of local frustration (latter) this complexity of connection will be removed. We would like to mention that spin echo between two single qubit has the equivalence of RVB state in FQHE and topological quantum computation with BP is responsible for the formation of higher states considering the Hall qubit at $\nu = 1$ as a building block of any QHE state.

V. HALL-QUBIT FORMATION IN FQHE THROUGH AHARONOV-BOHM PHASE

Two non-identical composite fermions residing in two consecutive Landau levels in FQHE as encircle each other, the relative Aharonov-Bhom (AB) type phase is developed. As the quasi-particles advance towards the edge of FQHE in a similar circular way, the developed current [36] should have a connection with this AB type phase in topological transportation.

The A-B interactions are the key source of forming two qubit Hall states identified as Hall qubit. Hence movement of Hall qubits would develop the A-B phase. In the physics of spin echo instead of Berry phase the incorporation of Aharonov-Bhom phase would be more appropriate as the rotation of qubits are equivalent to the rotation of fluxes around charges. If $e^{i\phi_s}$ be the Aharonov-Bohm phase between the two qubits for half period,incorporating spin-echo we find the antisymmetric Bell's state different from eq.(51) and (52)

$$|\Psi_{-}(t = \tau)\rangle = \frac{1}{\sqrt{2}}(e^{i\phi_s}|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - e^{-i\phi_s}|\downarrow\rangle_1 \otimes |\uparrow\rangle_2) \tag{76}$$

similar consideration for symmetric states

$$|\Psi_+(t=\tau)\rangle = \frac{1}{\sqrt{2}}(e^{-i\phi_s}|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 + e^{i\phi_s}|\downarrow\rangle_1 \otimes |\uparrow\rangle_2) \tag{77}$$

Splitting up these states and rearranging the symmetric and antisymmetric parts we have the doublet acquiring the matrix form of Aharonov-Bhom phase Υ as rotated from $t=0$ to $t=\tau$.

$$\begin{pmatrix} |\Psi_+\rangle \\ |\Psi_-\rangle \end{pmatrix}_\tau = \Upsilon \begin{pmatrix} |\Psi_+\rangle \\ |\Psi_-\rangle \end{pmatrix}_0 \tag{78}$$

where

$$\Upsilon = \begin{pmatrix} \cos\phi_s & -i\sin\phi_s \\ i\sin\phi_s & \cos\phi_s \end{pmatrix} = \cos 2\phi_s \tag{79}$$

This topological matrix phase Υ is developed from the Aharonov-Bohm phase ϕ_s as one qubit rotates around another. The qubits in QHE are quantized spinor having flux attached with charge. Their entanglement is equivalent to spin type echo where the topological phase dominates due to Aharonov-Bohm oscillation between them. This compel to change the Berry phase of the singlet state as in eq.(76) by the relative A-B type phase ϕ_s.

$$|\Phi_1(z)\rangle_\tau = e^{i\phi_s}|\Phi_1(z)\rangle_0 \tag{80}$$

This Hall qubit can be visualized in terms of entanglement of two oscillating qubits .

$$|\Phi_1(z)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \quad |\downarrow\rangle_1) \begin{pmatrix} 0 & -e^{-i\phi_s} \\ e^{i\phi_s} & 0 \end{pmatrix} \begin{pmatrix} |\uparrow\rangle_2 \\ |\downarrow\rangle_2 \end{pmatrix} \tag{81}$$

To form the singlet state between the qubits under A-B interactions in the spin echo method, the essential condition for antisymmetric QHE states are visualized by $\pm e^{i\phi_s} = \pm e^{i\pi} = \pm 1$.

In the formation of Hall qubit through the entangling of qubits in the different Landau level the A-B phase plays the key role in the spin type echo method. If the rotating qubits are in the same Landau level, the A-B phase changes to statistical phase. Considering the interaction [37] between two identical qubits in the same n^{th} Landau level for the composite particles filling factor $\nu = \frac{n}{2\mu_{eff}}$ the statistical phase becomes

$$\phi_s = exp \pm i\frac{n\pi}{2} \tag{82}$$

where for $n = 2,4,6...$ the change of statistics will be fermionic visualized by the phase $\phi_s = exp \pm i\pi$.On the other hand for $n = 1,3,5..$ the statistics will be bosonic $\phi_s = exp \pm i\pi/2$. It may be noted that for fractional filling factors the final statistics will be fermionic through proper combinations. With this view, the entanglement of two one qubits in the same Landau Level, the Hall qubit $|\Phi_1(z)\rangle$ will be formed when the exchange phase is $\pm i\pi$

$$|\Phi_1(z)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \quad |\downarrow\rangle_1) \begin{pmatrix} 0 & -e^{-i\pi} \\ e^{i\pi} & 0 \end{pmatrix} \begin{pmatrix} |\uparrow\rangle_2 \\ |\downarrow\rangle_2 \end{pmatrix} \tag{83}$$

Whenever the interaction takes place between dissimilar qubits in different Landau level the rotation of one against another develops the Aharonov-Bhom type phase that does not express their statistics. We assumed the transfer of the composite particle[19] from the inner edge in the n^{th} Landau level having filling factor ν_n picking up even integral $(2m)$ of flux ν_1 through the bulk of QH system and forming a new composite particle in the $(n+1)^{th}$ Landau level in the outer edge. The filling factor of the effective particle becomes $\nu_{eff} = \frac{n+1}{\mu_{eff}}$. The monopole strength μ_{eff} of the state ${\Phi_1}^{2m}\Phi_n$ can be considered as

$$\mu_{eff} = 2m\mu_1 + \mu_n. \tag{84}$$

Encircling of the composite particle in the inner edge having flux μ_n with charge q_n around the composite particle in the outer edge having corresponding flux μ_{eff} would develop a relative AB type phase

$$\phi_s = exp \pm \frac{i\pi}{2}(q_n\mu_{eff} + q_{eff}\mu_n) \tag{85}$$

In more simplified way it becomes

$$\phi_s \cong exp \pm \frac{i\pi}{2}[(n+\frac{1}{2}) - m\frac{\mu_1}{\mu_n}] \tag{86}$$

Since the concurrence $C = 1$ indicate the maximum entanglement and for disentanglement the value of minimum concurrence is $C = 0$, we can establish a relationship between the fluxes of the entangling qubits on the Hall surface [16].The maximum entanglement between the two quasi-particle results a relation between the respective two fluxes μ_1 and μ_n in terms of parent filling factor m and Landau level n.

$$\exp \pm \frac{i\pi}{2}[(n+\frac{1}{2}) - m\frac{\mu_1}{\mu_n}] = \exp \pm i\pi \tag{87}$$

This gives a ratio between the entangling fluxes μ_n and μ_1 in order to form the singlet pairs through AB oscillations in Quantum Hall effect.

$$\mu_n = \frac{2m}{2n-3}\mu_1 \tag{88}$$

It may be noted that for maximum entanglement $\mu_1 = 1$ results

$$\mu_n = \frac{2m}{2n-3}$$

and for minimum entanglement both μ_1 and μ_n becomes zero.

The physics behind the formation of higher Hall states take place through the entanglement of Hall qubits $|\Phi(z)_1>$ in the lowest landau level. Here the A-B phase or statistical phase plays the key role in the process of spin echo with the essential condition $\pm e^{i\phi_s} = \pm e^{i\pi} = \pm 1$. The outcome of entanglement of two Hall qubits is

$$|\alpha(z)> = <\Phi_1(z)|\begin{pmatrix} 0 & -e^{-i\pi} \\ e^{i\pi} & 0 \end{pmatrix}|\Phi_1(z)> = \begin{pmatrix} 0 & \Phi_1(z) \\ -\Phi_1(z) & 0 \end{pmatrix} \tag{89}$$

where $\Phi_1(z) = (u_i v_j - v_i u_j)$. On the similar manner we realize that the entanglement of two $|\alpha(z)>$ gives rise the state formed by the square of Hall qubits.

$$|\gamma(z)> = <\alpha(z)|\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}|\alpha(z)> = \begin{pmatrix} 0 & {\Phi_1}^2(z) \\ -{\Phi_1}^2(z) & 0 \end{pmatrix} \tag{90}$$

In order to maintain the antisymmetric nature of the Hall state the power of the Hall qubit should be odd. This is possible only when two asymmetric Hall qubits (one even power with another odd power) entangle under topological interactions

$$<\gamma(z)|\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}|\alpha(z)> = \begin{pmatrix} 0 & {\Phi_1}^3(z) \\ -{\Phi_1}^3(z) & 0 \end{pmatrix} \tag{91}$$

forming a Hall state in the parent Landau filling factor $\nu = m = odd$.

We like to conclude mentioning that the hierarchical FQHE states are formed through the process of quantum teleportation. If there are three entities defined by 1,2, and 3, then transportation of 1 to 3 through 2 will be

$$|\Psi>_{123} = |\Phi_1>|\Psi_{23}> \tag{92}$$

$$= \frac{1}{2}(1+\sigma^1.\sigma^3)|\Psi_{12}>|\Phi_3> \tag{93}$$

Similar reflection of quantum teleportation [16] in FQHE motivate us to write

$$\Psi_\nu{}^m = {\Phi_1}^{2m}\Phi_n = {\Phi_1}^{2m}{\Phi_1}^{\frac{1}{n}} = \frac{1}{2}(1+\sigma^1.\sigma^3)|{\Phi_1}^{\frac{1}{n}}{\Phi_1}^{2m} \tag{94}$$

a hierarchical FQHE state whose extensive study through the entanglement of Hall qubits maintaining the antisymmetric nature of the exchange phase is to be done in future.

VI. DISCUSSION

The quantized spinor residing on the surface of frustrated sphere is celebrating a new kind of Berry phase. The un-frustrated Berry phase, the usual solid angle may be recovered under special condition. This frustrated Berry phase could be the key source of resolving various physical problems in disorder system. The Hall state for the lowest Landau level at $\nu = 1$ is highly frustrated. They are the singlet states identified as the Hall qubit, the building block of other higher IQHE/FQHE states at different filling factors. Since these spin singlet states forming a RVB gas is equivalent to fractional quantum Hall fluid, the description of background Physics through quantum computation will be of ample interest. In this paper we have studied the Physics behind the Hall qubit formed by entanglement of two qubits where one qubit is rotating in the field of the other with Berry phase or Aharonov-Bohm phase in the respective lowest and Higher Hall states. Image of spin echo has been reflected in the formation of Hall qubit. These states have matrix Berry phase which are responsible for pumped charge to flow and acts as a local order parameter of singlet states. Further we pointed out that the antisymmetric nature of $\nu = 1/m$ FQHE states depend on their acquired Berry phase. With the condition of concurrence for maximum entanglement, a proper ratio between the fluxes of the entangling qubits has been evaluated. At the end, it has been mentioned that the states in hierarchies of FQHE can be studied in the light of quantum teleportation whose extensive study will be of new interest in future.

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